[5]Zeiberg, S.I., "Transition Correlations for Hypersonic Wakes," *AIAA Journal*, Vol. 2, March 1964, pp. 564-565.
[6]Lyons, W.C., Brady, J.J., and Levensteins, Z.J., "Hypersonic Drag, Stability, and Wake Data for Cones and Spheres," *AIAA Journal*, Vol. 2, March 1964, pp. 564-565.
[6]Lyons, W.C., Brady, J.H., and Levensteins, Z.J., "Hypersonic Drag, Stability, and Wake Data for Cones and Spheres," *AIAA Journal*, Vol. 2, Nov. 1964, pp. 1948-1956.
[7]Levensteins, Z.J., and Krumins, M.V., "Aerodynamic Characteristics of Hypersonic Wakes," *AIAA Journal*, Vol. 5, Sept. 1967, pp. 1596-1602.
[8]Slattery, R.E. and Clay, W.G., "The Turbulent Wake of Hypersonic Bodies," Paper presented at American Rocket Society 17th Annual Meeting, Los Angeles, California, Nov. 13-18, 1962.
[9]Wilson, L.N., "Body Shape Effects on Axisymmetric Wakes," GM Defense Research Laboratories TR64-02K, Oct. 1964.
[10]Wilson, L.N., Private communication.
[11]Pallone, A, Erdos, J., and Eckerman, J., "Hypersonic Laminar Wake Transition Studies," *AIAA Journal*, Vol. 2, May 1964, pp. 855-863.
[12]Sato, H. and Okada, O., "The Stability and Transition of an Axisymmetric Wake," *Journal of Fluid Mechanics*, Vol. 26, Part 2, 1966, pp. 237-253.
[13]Demetriades, A. and Gold, H., "Transition to Turbulence in the Hypersonic Wake of Blunt-Bluff Bodies," *American Rocket Society Journal*, Vol. 32 Sept. 1962, pp. 1420-1421.
[14]Demetriades, A., "Hot-Wire Measurements in the Hypersonic Wakes of Slender Bodies," *AIAA Journal*, Vol. 2, Feb. 1964, pp. 295-250.
[15]Kohrs, R., Rannabecker, C., Patay, S., and Wells, D., "The Determination of Hypersonic Drag Coefficients for Cones, Biconics, and Triconics," Avco Missile Systems Division AVMSD-0136-67-CR, March 1967.
[16]Waldbusser, E., "Hypersonic Laminar Near Wakes," General Electric Reentry Systems Department Document No. 68SD274, June 1968.

Deformations and Thermal Conductances of Cone to Flat Contacts

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Introduction

IN many engineering situations it is important to control and to be able to predict the thermal conductance of metallic joints. This is often difficult as there are several interrelated parameters which cannot be specified accurately. One of the main requirements is a knowledge of the contact geometry, viz. the sizes and shapes of the small actual contact spots, and how these are distributed over the apparent contact zone. Mathematical models of the contact zone usually specify a contact spot, for which the potential heat flowfield may be determined. The main purpose of this Note is to compare the predicted and measured thermal conductances of single contact spots of easily controlled shapes, viz. cones pressing against plane surfaces, for which the actual contact areas may be measured accurately. Another purpose is the determination of actual support pressure of cone to flat contacts over a wide range of cone angles, as existing data are incomplete.

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Literature Survey

Geometrically simple models of contacts have been examined mathematically and physically in several studies of thermal conductance, e.g. Refs. 1-3, and now form the main bases of computer-aided predictions of the thermal behavior of metallic joints. In all these predictions it is necessary to specify an appropriate "hardness" (or true contact pressure) of the materials in contact, accounting for both elastic and plastic modes of deformation, and peculiarities of shape. The heat-transfer tests and contact area measurements reported in Ref. 4 were sufficiently anomalous to provoke the investigation reported in this paper, using very simple shapes of contact elements, and joint pairs of the same material.

Theory

Cone Deformation

Although the plastic behavior of metallic cone to flat contacts under compression has been studied since 1926,[5,6] there is still no general analytical solution available for the prediction of the shape of a cone being flattened against a plane. However, the behavior of the two-dimensional wedge of rigid-plastic material deforming against a smooth plane is well established,[6] and is used herein as the basis of an approximate slip-line field solution for the cone problem, from which the deformed shape and the true contact pressure may be determined. Observations made during deformation tests indicated that the predicted and measured shapes matched closely.

Referring to Fig. 1, assuming incompressible material, and a deformation mode in which lengths m and n are equal, the relationship between the cone semiangles β and the slip angle α can be shown to be

$$\tan\beta = \frac{(1+\sin\alpha)^3}{\cos\alpha\,(3+3\sin\alpha+\sin^2\alpha)}$$

This equation is shown plotted on Fig. 2, which also shows measured angles of cones of mild steel and of aluminium deforming against a smooth hard surface (made of tool steel). For large cone angles, α is almost equal to β.

Thermal Conductance

The temperature distribution in a homogeneous and isotropic conductor during steady-state heat flow conditions is described by Laplace's equation. For axisymmetric

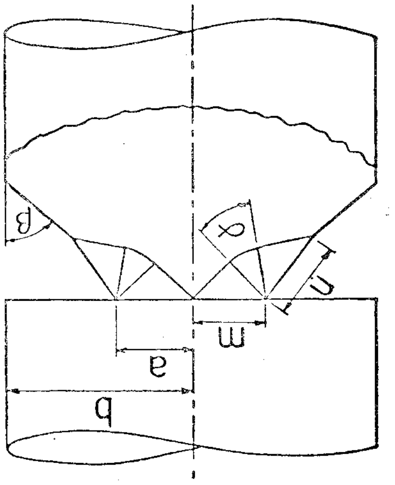

Fig. 1 Cone to flat joint showing slip-line field in deforming cone.

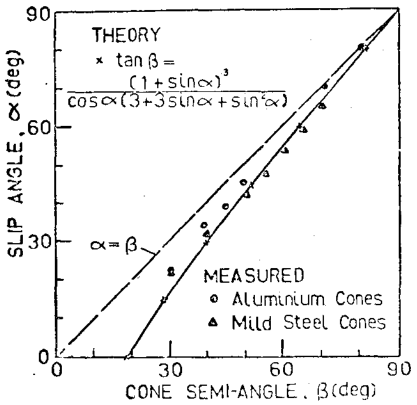

Fig. 2 Relation between cone and slip angles.

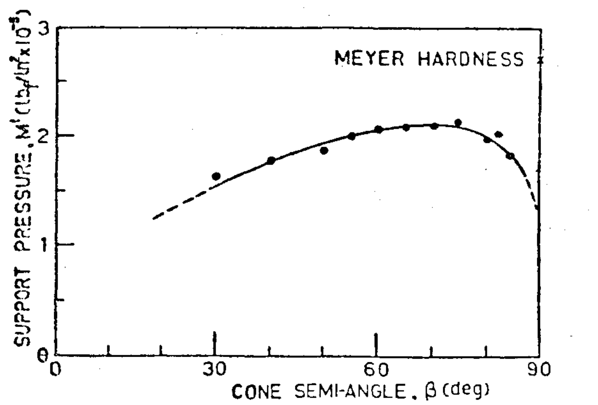

Fig. 4 Variation of support pressure with cone angle for mild steel cones.

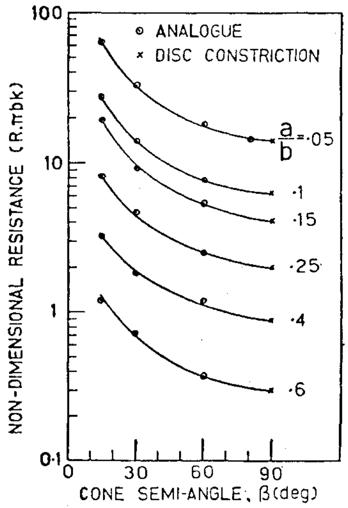

Fig. 3 Thermal resistance of deformed cones.

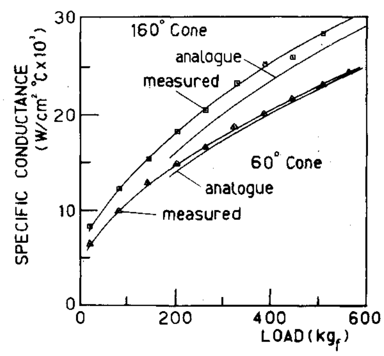

Fig. 5 Specific thermal conductance of cone to flat joints.

distributions this equation in cylindrical polar coordinates is

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}=0$$

Solutions of this to fit specified boundary conditions may be obtained using analytical, numerical, and analogue methods, the latter two being attractive when complicated shapes are under examination.

Figure 3 shows the theoretical variation of the thermal resistances of deformed cones over a wide range of cone semiangles β and minor to major radius ratios a/b. This nondimensional form of resistance (viz. thermal resistance R multiplied by πbk) allows simple presentation of results for comparison with published data. The solid curves of Fig. 3 were derived from electrical analogue measurements using liquid electrolyte in perspex models of deformed cones; solutions obtained by computation using finite difference techniques are in close agreement with those from the analog.

Experimental Work

Cone Deformation

Sharply tipped conical test pieces of mild steel and aluminum made from 0.75 in. diam bar were loaded axially against a smooth plane platten of tool steel. After each increment of axial load the contact shapes were measured whilst still loaded, using a traversing microscope viewing from two directions mutually perpendicular. At the end of the compression test the deformed specimens were examined under a large magnification profile projector, to obtain values for α, a, and the length of the deformed tip. This subsequent examination was made so as to determine any differences between the loaded conditions (including both elastic and plastic deformations) and the subsequent unloaded conditions (only plastic deformations being present). There were no detectable differences between these two, for the present tests on 18 specimens.

Figure 2 shows the measured deformations, and Fig. 4 shows the measured variations of support pressure M^1 (the applied load divided by the measured contact area). For each cone angle, with a given material, the value of M^1 was found to be consistent to $\pm 3\%$ of a mean value over the whole range of applied load apart from very lightly loaded conditions. The reduction in support pressure at angles above 80° should be noted, as this is an extension of published data and was not predicted.[7]

Thermal Conductance Measurements

Duplicate test pieces with thermocouples fitted were used for thermal measurements in a column type rig inside an evacuated chamber (typically a pressure of 0.1 Torr was used). Thermocouple readings during steady-state heating conditions were processed by computer to give the specific thermal conductance of the joint at each loading. the results are shown in Fig. 5. Typical values of extrapolated temperature drop at the interface were 50-60°C, probably accurate to ± 3°C.

Discussion of Results

Cone Deformation

The assumption that m and n of Fig. 1 are equal is verified by the measurements herein, for specimens of mild steel or aluminum. Work hardening effects were not examined; elastic recovery of a deformed tip was negligible in the computation of contact area. The derived values of support pressure were consistent for individual specimens, and for each material, the behaviors of specimens covering a wide range of cone angles were well ordered. However, this latter behavior extends the range of published data and conflicts with extrapolations.[7]

Thermal Conductance

Figure 5 shows the variation of specific conductance of the cone to flat joints with applied load, for two different cone angles and two different combinations of joint materials. In all cases, the agreements between the measured thermal conductances and predicted values (based on analog or numerical solutions) are good. In each case, the support pressure was taken from physical testing of the material, typically as shown in Fig. 4. The thermal testing indicated no effect of strain hardening changing the local thermal conductivity of the contact materials.

Conclusions

As a result of the physical and thermal tests on compressed cone to flat metallic contacts it is possible to predict the thermal conductance of this type of joint with confidence. Parameters required for these predictions include the shape of the deformed joint whilst loaded, and the true support pressure. The latter varies with the initial cone angle, and is always less than the Meyer Hardness of the material of the conical element. The anomalies of previous work[4] based on simpler physical models and on assumed values of support pressure equal to the Meyer Hardness have been resolved.

The electrical analogue proved particularly useful for rapid determinations of joint conductances of many shapes of contact, giving good agreement with computer predictions based on constant physical properties of the joint materials. Both of these methods produced results in good agreement with thermal conductance measurements, for a variety of cone angles and joint materials.

References

[1] Cetinkale, T. N. and Fishenden, M., "Thermal Conductance of Metallic Surfaces in Contact," General Discussion on Heat Transfer, I. Mech. E. London, 1951, pp. 271-275.

[2] Clausing, A. M. and Chao, B. T., "Thermal Contact Resistance in a Vacuum Environment," *Journal of Heat Transfer,* ASME, Ser. C, Vol. 87, 1965, pp. 243-251.

[3] Kitsha, W. W. and Yovanovich, M. M., "Experimental Investigation on the Overall Thermal Resistance of Sphere – Flat Contacts," *AIAA Progress in Astronautics and Aeronautics: Heat Transfer with Thermal Control Applications,* Vol. 39, edited by M. Michael Yovanovich, MIT Press, Cambridge, Mass., 1975, pp. 93-110.

[4] Williams, A., "Heat Flow Through Single Points of Metallic Contacts of Simple Shapes," *AIAA Progress in Astronautics and Aeronautics: Heat Transfer with Thermal Control Applications,* Vol. 39, edited by M. Michael Yovanovich, MIT Press, Cambridge, Mass., 1975, pp. 129-142.

[5] Mallock, A., "Hardness," *Nature,* No. 2934, Vol. 117, 1926, pp. 117-118.

[6] Hill, R., *The Mathematical Theory of Plasticity,* Oxford University Press, N.Y., 1967.

[7] Bowden, F. P. and Tabor, D., *The Friction and Lubrication of Solids,* Part 1, Oxford University Press, N.Y., 1950.

Interaction Effects of Multiple Cracks

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I. Introduction

THE application of fracture mechanics to fatigue-crack growth and residual-strength analyses has resulted in much progress during the last decade. Yet the presence of cracks in engineering structures still poses many serious research problems that have to be solved. One such problem is the interaction of multiple cracks originating at fastener holes.

Fatigue cracks usually originate in the regions of high stress concentration, which exist notwithstanding careful detail-design procedures. Hardly any assembled structure, such as fastener holes and access holes, is free of geometric discontinuities. A review of U.S. Air Force aircraft structural failures[1] revealed that cracks emanating from fastener holes represent the most common origin of these failures.

To date, there has been much useful work done on the problem of determining reliable stress-intensity factors for cracks emanating from fastener holes. Almost all of these analytical determinations are based upon modifications of a solution obtained by Bowie[2] for cracks emanating from a circular hole in an infinite elastic sheet. The problem of the interaction effects of periodic parallel straight cracks having the same crack length has been investigated.[3,4] But the interaction effects of cracks originating at a multiplicity of fastener holes has not been sufficiently studied.

In order to minimize the test time and cost, and to gather the maximum amount of useful data, specimens containing multiple cracks have been extensively employed in experimental fracture mechanics programs. Unless the crack lengths are small and/or the spacing between cracks is relatively large, interaction between cracks will occur, and the data generated under such test, if not meaningless, may be very difficult to analyze because current analytical methodology is not able to account for such an interaction. The purpose of this study was to investigate the effect of the additional cracked hole or holes on the stress-intensity factor of a primary crack.

II. Analysis and Discussion

To investigate the interaction effects between the cracks emanating from fastener holes, the finite-element method was used to compute the stress-intensity factors for the typical cracked geometry shown in Fig. 1. Due to symmetry, it was necessary to model only the first quadrant of the plate. A detail of the two-dimensional finite-element model is shown in Fig. 2. The model consists of 325 nodes and 525 constant-strain triangular elements. The crack-tip regions in the finite-element model were represented by eight-node and ten-node singularity elements developed at Lockheed-Georgia[5] for the purpose of computing the stress-intensity factors appropriate to various crack lengths. A detailed discussion of these singularity elements is given in Ref. 5.

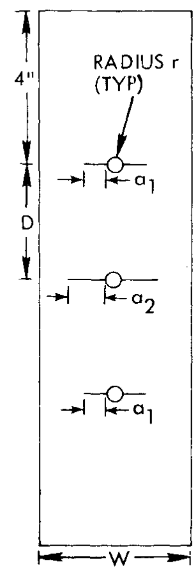

Fig. 1 Specimen geometry.

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